Contest Design with Interim Types

Matthew W. Thomas*

March 6, 2025

Abstract

A principal may know the interim distribution of agent types rather than the ex-ante distribution. For example, she may have information about types but not be permitted to discriminate due to anonymity or legal restrictions. This setting is rarely studied in mechanism design because full surplus extraction is trivial. However, this setting is frequently studied in contest design where functional form assumptions prevent trivial results. We model contest design as a general allocation rule without any functional form assumptions. Instead, we impose efficiency, the requirement that the entire prize budget must be allocated in response to any bid profile. This condition holds in all popular contest forms. We find that efficiency is sufficient to prevent full surplus extraction when there is only one marginal player. In the two-player case, the overall optimal contest is one of two popular models: an all-pay auction with bid caps when heterogeneity is low or a difference-form contest when heterogeneity is high.

1 Introduction

Contests are models of conflict in which risk neutral players exert costly effort to win a prize. They are commonly used to model interactions such as competition among employees for a promotion, athletic competitions, and political campaigns for office. When one player is more capable than another, contests become less competitive. This decreases total effort.

*Federal Trade Commission and Economic Research Organization, mthomas@ftc.gov, The views expressed in this article are those of the author and do not necessarily reflect those of the Federal Trade Commission or any individual Commissioner.

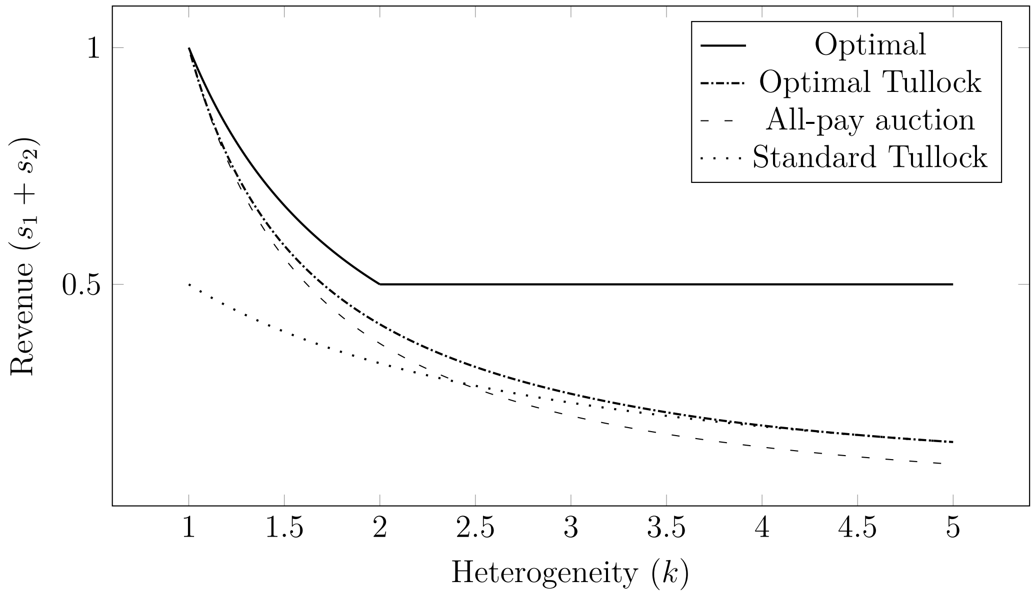

Figure 1: Equilibrium revenue is plotted for different contests at varying levels of player heterogeneity. The selectivity of the Tullock contest can be adjusted for each level of heterogeneity to increase revenue. The overall optimal contest achieves much more revenue at each level of heterogeneity.

A principal who stands to gain from the efforts of participants would want to design the contest to maximize these efforts. It is a common exercise to restrict the principal to a particular functional form and assume that she has some deep knowledge about the types of the players. If the principal can discriminate directly (e.g., Ewerhart, 2017; Franke, Leininger, et al., 2018; Nti, 2004), then the principal is assumed to have complete information. If players cannot discriminate directly (e.g., Fang, 2002; Franke, Kanzow, et al., 2014), then the principal is assumed to know the interim distribution of types. That is, the empirical type distribution.

There are many settings where it is reasonable to assume that the principal knows the interim distribution of types. For example, the principal may have anonymized data or could have access to complete data but be unable to discriminate directly due to legal restrictions. In particular, there are settings where it is unacceptable to bias a competition in favor of weaker players. For example, giving a less productive employee an advantage in a labor tournament may be seen as rewarding bad performance. This paper is primarily concerned with determining the optimal contest for a principal who knows the interim type distribution without imposing functional form restrictions.

I instead impose the restriction that the contest must be anonymous and efficient.

Without any restrictions, the problem of designing a contest or mechanism with interim types is trivial. The principal can obtain complete information – and thus extract the full surplus – by refusing to allocate the prizes unless the reports exactly match the known distribution of interim types. For example, in a standard monopoly sale problem with two players and one prize, the principal can sell the good for a price equal to the maximum of the two players' valuations.

This, however, assumes that the principal can withhold the prize when the reports do not match. In many applications, this is unreasonable. Players competing for promotions or public office are running for a seat which often must be filled. In a sporting event, declaring that all teams lose is generally unacceptable.

I find that efficiency, the condition that all prizes must be allocated in response to all scores, is sufficient to meaningfully constrain the principal's surplus extraction if and only if there are $n-1$ prizes to allocate (i.e., one less than the number of players). In the case with two players and one prize, the optimal contest can be implemented with either an all-pay auction with bid caps when heterogeneity is low or a difference-form contest when heterogeneity is high (revenue pictured in Figure 1). In the general case, with n players and $n-1$ prizes, the optimal mechanism shares many of the features of the two-player case. The form of the mechanism depends on the differences in ability between the weakest player and the others.

Because the stronger players are more productive than weaker players, the socially optimal outcome does not involve any effort from the weakest player. However, effort from the weakest player relaxes the incentive compatibility conditions of stronger players. The all-pay auction with bid cap form is used when the principal finds it optimal to demand effort from the weakest player. If the weakest player is sufficiently unproductive relative to the second weakest player (more than n times as unproductive) the principal will not have the weaker player exert effort.

From a mechanism design perspective, the designer's main question is whether the lowest type should be pooled together with other types on the bottom. If not, it is suboptimal to allocate any share of a prize to the lowest type. In the absence of any reward, the effort demanded of the weakest player must be zero.

This work takes a similar approach to Letina et al. (2020). However, unlike that study, we consider heterogeneous agents and do not allow the principal to engage in direct discrimination or modify/withhold the prize. This work on optimal contest design contributes to a literature on revenue dominance in anonymous, efficient contests

(Fang, 2002; Franke, Kanzow, et al., 2014) by characterizing an anonymous, efficient contest which weakly dominates all others and contributes to a much larger literature on contest design.

2 Model

We begin with a two-player model with a single prize. The extension with more players and prizes is explored in Section 4. The two players simultaneously choose scores $s_1, s_2 \geq 0$ to compete for a prize. The designer selects a contest success function which is a function that maps scores to the probability of each player winning the prize. In particular, the probability of Player i winning the prize in the contest when she chooses score s_i and her opponent chooses score s_{-i} is:

$$p_i(s_i, s_{-i}).$$

The prize of the contest is worth one to both players. So, the two players have the same value. However, their scores have different costs. In particular, the score is $k > 1$ times more costly for Player 2 than for Player 1. One interpretation of this is that it takes Player 1 less effort to produce a high score because she is more skilled. This is equivalent to a model where the two players value the prize differently.[1]

The final payoffs are:

$$U_1(s_1, s_2) = p_1(s_1, s_2) - s_1$$
$$U_2(s_1, s_2) = p_2(s_2, s_1) - ks_2.$$

The contest designer selects p_1, p_2 to maximize the equilibrium expected revenue, $E[s_1 + s_2]$. As is standard in the mechanism design literature, the equilibrium of the contest need not be unique. We assume that the designer can select her most preferred equilibrium.

We impose two restrictions on the contest success function that the designer may choose:

Definition 1 (Anonymous)**.** $p_1(x, y) = p_2(x, y)$ for all $x, y \geq 0$.

[1] It is also equivalent to one where players face some nonlinear monotone cost function, c. However, the objective of the principal would be to maximize $E[c(s_1) + c(s_2)]$ instead of $E[s_1 + s_2]$.

Definition 2 (Efficient). $p_1(x, y) + p_2(y, x) = 1$ for all $x, y \geq 0$.

Anonymity means that both players are treated the same by the designer. Efficiency means that someone is always a winner. We will see that these conditions do not individually constrain the designer's revenue. However, when the two conditions are imposed together, the designer is meaningfully constrained.

3 Contest Design

It is easy to see that under the assumptions of anonymity and efficiency, the designer cannot extract the full surplus. To see this, note that the unique first best score profile is $s_1 = 1$ and $s_2 = 0$. So, both players receive a payoff of zero in the first best, but Player 1 can choose zero instead of one to receive a payoff of $p(0, 0) = 0.5$.

Note that this alone does not imply that the designer is *meaningfully* constrained. It could be that the designer can extract some quantity arbitrarily close to the the full surplus. In fact, I demonstrate in Section 4 that this is possible with n players and less than $n - 1$ prizes.

However, in the two-player case, I will show that although the designer is not constrained by either of the two conditions alone, she is meaningfully constrained by the two conditions together.

3.1 Anonymity Without Efficiency

If we allow for inefficiency, the designer can achieve the first best. For example, the following all-pay auction with a reserve bid achieves the first best:

$$p(x, y) = \begin{cases} 0 & \text{if } x < \max(y, 1) \\ \frac{1}{2} & \text{if } x \geq 1, x = y \\ 1 & \text{if } x \geq 1, x > y. \end{cases}$$

This is because Player 1 is willing to meet the reserve bid of 1 and Player 2 is not.

One might wonder if the designer can still extract the full surplus with a smaller reserve bid. The answer is yes. In fact, $p(0, 0) = 0$ is sufficient for the first best to be attainable under anonymity. That is, the designer need only be able to withhold the prize when both players play zero. To see this, consider the following contest which satisfies both properties except at zero.

$$p(x,y) = \begin{cases} 1 & \text{if } x - y > 1 \\ x - y & \text{if } x - y \in (0,1] \\ 0 & \text{if } x = y = 0 \\ \frac{1}{2} & \text{if } x = y \neq 0 \\ 1 + x - y & \text{if } x - y < 0. \end{cases}$$

This contest has an equilibrium at the first best yet only denies the prize to players when they both exert no effort.

3.2 Efficiency Without Anonymity

Direct discrimination is similar to a reserve bid. It is possible to mimic any reserve bid through direct discrimination by promising the prize to the weaker player whenever the reserve is not met. In particular, take either example from the previous section and set $p_1(x,y) = p(x,y)$ and $p_2(y,x) = 1 - p(x,y)$. Such a contest is efficient and will be strategically equivalent to the original contest. For example, consider the aforementioned all-pay auction with a reserve bid. With this transformation,

$$p_1(x,y) = \begin{cases} 0 & \text{if } x < \max(y,1) \\ \frac{1}{2} & \text{if } x \geq 1, x = y \\ 1 & \text{if } x \geq 1, x > y \end{cases}$$

$$p_2(x,y) = \begin{cases} 0 & \text{if } y \geq 1, y > x \\ \frac{1}{2} & \text{if } y \geq 1, x = y \\ 1 & \text{if } y < \max(x,1). \end{cases}$$

This contest has an equilibrium at the first best. For Player 1, this is equivalent to an all-pay auction with a reserve bid because the prize is allocated to Player 2 whenever the reserve is not met.

3.3 Results of Constrained Contest Design

I now restrict the principal to use an anonymous, efficient contest. The optimal contest depends on the level of heterogeneity, k.

Theorem 1. *If $k \geq 2$, the optimal anonymous, efficient contest obtains revenue $\frac{1}{2}$. This can be implemented with the following difference-form contest:*

$$p(x,y) = \begin{cases} 1 & \text{if } x-y > \frac{1}{2} \\ \frac{1}{2} + x - y & \text{if } x - y \in \left[-\frac{1}{2}, \frac{1}{2}\right] \\ 0 & \text{if } x - y < -\frac{1}{2}. \end{cases}$$

Theorem 2. *If $k \leq 2$, the optimal anonymous, efficient contest obtains revenue $\frac{1}{k}$. This can be implemented with the following all-pay auction with bid caps:*

$$p(x,y) = \begin{cases} 1 & \text{if } \frac{1}{2k} \geq x > y \text{ or } y > \frac{1}{2k} \\ \frac{1}{2} & \text{if } x = y \\ 0 & \text{if } \frac{1}{2k} \geq y > x \text{ or } x > \frac{1}{2k}. \end{cases}$$

The revenue of the overall optimal contests is plotted in Figure 1. The proof of the two-player case is instructive. So, I include the proof in the body of the text. The proof consists of proving Lemma 1 which establishes a bound on the revenue. Given this bound, we need only verify that the contests in Theorems 2 and 1 attain this bound.

Lemma 1. *In any equilibrium of a anonymous, efficient contest,*

$$E[s_1 + s_2] \leq \begin{cases} \frac{1}{k} & \text{if } k < 2 \\ \frac{1}{2} & \text{if } k \geq 2. \end{cases}$$

Proof. In equilibrium, each player must weakly prefer her equilibrium payoff over copying the strategy of her opponent. Therefore, the following weak *incentive compatibility* condition is necessary for equilibrium:

$$\frac{1}{2} - E[s_2] \leq E[U_1(s_1, s_2)].$$

Together with $E[U_2(s_1, s_2)] \geq 0$ this implies

$$\frac{1}{2} - E[s_2] \leq E[U_1(s_1, s_2) + U_2(s_1, s_2)]$$
$$\frac{1}{2} - E[s_2] \leq 1 - E[s_1 + ks_2]$$

where the last line follows from efficiency. Rearranging this equation gives an upper bound on the revenue:

$$E[s_1 + s_2] \leq \frac{1}{2} + (2 - k)E[s_2]$$

We will refine this bound to

$$E[s_1 + s_2] \leq \begin{cases} \frac{1}{k} & \text{if } k < 2 \\ \frac{1}{2} & \text{if } k \geq 2 \end{cases}$$

by showing that $E[s_2] \leq \frac{1}{2k}$ because Player 2 cannot win with probability more than one half. This can be derived directly from the same incentive compatibility conditions that we used to construct the bound. Player 2 cannot have a strict incentive to copy the strategy of Player 1. Therefore,

$$\frac{1}{2} - kE[s_1] \leq E[p(s_2, s_1) - ks_2]$$
$$E[s_2] \leq E[s_1] + (k)^{-1}\left(E[p(s_2, s_1)] - \frac{1}{2}\right).$$

Similarly, the incentive compatibility condition for Player 1 implies,

$$\frac{1}{2} - E[s_2] \leq E[p(s_1, s_2) - s_1]$$
$$E[s_2] \geq E[s_1] - \left(E[p(s_2, s_1)] - \frac{1}{2}\right).$$

Combining these two inequalities and $k \geq 1$ yields $E[p(s_2, s_1)] \leq \frac{1}{2}$. □

Case 1: $k < 2$. This means that revenue cannot exceed $\frac{1}{k}$. We can reach this upper bound with an all-pay auction with a bid cap at $\frac{1}{2k}$ as in Che and Gale (1998).

That is

$$p(x,y) = \begin{cases} 1 & \text{if } \frac{1}{2k} \geq x > y \text{ or } y > \frac{1}{2k} \\ \frac{1}{2} & \text{if } x = y \\ 0 & \text{if } \frac{1}{2k} \geq y > x \text{ or } x > \frac{1}{2k}. \end{cases}$$

This has an equilibrium at $s_1 = s_2 = \frac{1}{2k}$ which achieves the upper bound for $k < 2$.

Case 2: $k \geq 2$. The above implies that revenue cannot exceed one half. Consider the following difference-form contest as in Che and Gale (2000):

$$p(x,y) = \begin{cases} 1 & \text{if } x - y > \frac{1}{2} \\ \frac{1}{2} + x - y & \text{if } x - y \in \left[-\frac{1}{2}, \frac{1}{2}\right] \\ 0 & \text{if } x - y < -\frac{1}{2} \end{cases}$$

The above contest has an equilibrium at $s_1 = \frac{1}{2}$ and $s_2 = 0$ which achieves the upper bound for $k \geq 2$.

Therefore, the bound is obtained in each case by the contests defined in in Theorems 2 and 1. Intuitively, the core tension of this optimization problem is that the stronger player can imitate the weakest player. Given this, designing a contest that requires effort from the weakest player has two competing effects.

On the one hand, the weakest player is less productive than the stronger player. So, using a share of the prize to demand effort from the weakest player uses resources that could be used to obtain effort from the strongest player. On the other hand, obtaining effort from the weakest player turns up competition on the stronger player by making the weakest player more costly to imitate. That is, it loosens the incentive compatibility constraint.

4 More than two players

We can extend the result to find the optimal contest with n players and $n-1$ prizes where each player can receive at most one prize. Other cases where there is more than one loser or where multiple prizes may be allocated to the same player have trivial solutions where almost all the surplus is extracted.

Lemma 2. *Suppose there are $n \geq 3$ players. For any $\epsilon > 0$ and $m < n-1$, there exists an anonymous, efficient contest with m prizes such that total effort is $\left(\sum_{i=1}^{m} \frac{1}{k_i}\right) - \epsilon$. That is, the principal extracts the maximum surplus minus epsilon.*

To see this, consider the case with one prize and three players with costs $1, k_2, k_3$. The designer can impose a contest where the score profile $(1-\epsilon, \epsilon/k_2, 0)$ results in the prize being allocated with probabilities $(1-\epsilon, \epsilon, 0)$. This is an equilibrium if the designer punishes deviations by awarding the prize to a player who did not deviate. A single player can deviate from this profile either by copying another player's score or by playing another unique score. Either way, at least one of the non-deviating players can be identified by their score, and the designer can award the prize to that player.

Note that this method does not work when there are two prizes. In such a case, if Player 1 imitates Player 3 by playing a score of 0, the designer must still award a prize with 50% probability to each of Player 1 and Player 3 because there are two prizes to allocate and only one player who can be identified as non-deviating.

The *epsilon* term in Lemma 2 is an artifact of the contest formulation where players are identified by their effort alone rather than through some revelation mechanism. That is, Player 2 and Player 3 cannot both play zero because the principal would be unable to tell the difference between them even when they do not deviate. I dispense with this in the remainder of this section and instead consider optimal mechanisms.

4.1 More than two players with one marginal player

Because the optimal anonymous, efficient contests with n players and less than $n-1$ prizes are trivial, I focus on the case with n players and $n-1$ identical prizes. I will show that the optimal contest can be implemented with a piecewise version of the two-player case. The form of the contest depends on the difference in ability between the weakest player and the others.

Proposition 1. *If there are n players, $n-1$ prizes, and $\frac{k_n}{k_{n-1}} \geq n$, then in the optimal anonymous, efficient mechanism, the weakest player exerts no effort and the principal obtains revenue*

$$\sum_{i=1}^{n-1} \frac{1}{2k_i}.$$

Proposition 2. *If there are n players, $n-1$ prizes, and $\frac{k_n}{k_{n-1}} \leq n$, then in the optimal anonymous, efficient mechanism, all players exert effort, the principal obtains revenue less than*

$$\frac{1}{k_n} + \sum_{i=1}^{n-2} \frac{1}{k_i},$$

and at least the two weakest players exert the same effort at the bottom.

Propositions 1 and 2 are the first two steps in a longer algorithm for finding the optimal contest with n players. The algorithm can be found in Appendix B. These two steps are the only ones that exist in the two-player case. Recall that in Section 3, $k_1 = 1$ and $k_2 = k$. So, when $n = 2$, Proposition 1 is consistent with Theorem 1, and Proposition 2 is consistent with Theorem 2.

As in the two-player case, the mechanism used depends on whether the principal finds it optimal to demand effort from the weakest player. If the weakest player (n) is sufficiently productive relative to the second weakest player ($n-1$), the principal will have Player n and Player $n-1$ exert the same effort. This is the case in Proposition 2. Otherwise, the principal will have Player n exert no effort as is the case in Proposition 1.

These two weakest players behave in much the same way that the two players in the two-player case behave. The main difference is that the score of the weakest player affects the incentive compatibility constraints of the other players. As a result, there is more upside to demanding effort from the weakest player. In particular, the principal strictly prefers to demand effort from the weakest player if and only if $n > \frac{k_n}{k_{n-1}}$. So, there is a tendency to demand full participation in contests with more players.

4.2 Three Players and Two Prizes

The problem with more than two players introduces complexity that does not exist in the two-player case. However, the three-player case captures all the features of the general problem. I use the case with three players and two prizes to demonstrate these features. The full algorithm for the general case can be found in Appendix B.

Theorem 3. *If there are three players and two prizes, the optimal anonymous, effi-*

cient mechanism obtains revenue

$$\begin{cases} \frac{1}{2k_1} + \frac{1}{2k_2} & \textit{if } \frac{k_3}{k_2} \geq 3 \\ \frac{1}{2k_1} + \frac{3}{2k_3} & \textit{if } \frac{k_3}{k_2} \leq 3 \leq \frac{k_3}{k_1} \\ \frac{3}{k_3} & \textit{if } \frac{k_3}{k_1} \leq 3 \textit{ and } \frac{k_3}{k_2} \geq 2 \\ \frac{1}{k_1} + \frac{3 - k_3/k_1}{2k_2} & \textit{if } \frac{k_3}{k_1} \leq 3 \leq \frac{k_2+k_3}{k_1} \textit{ and } \frac{k_3}{k_2} \leq 2 \\ \frac{6 - \frac{k_2+k_3}{k_1}}{2k_1} & \textit{if } \frac{k_2+k_3}{k_1} \leq 3 \textit{ and } \frac{k_3}{k_2} \leq 2. \end{cases}$$

The first line in the piecewise expression of Theorem 3 is the same as the revenue in Proposition 1. In this case, the weakest player exerts no effort and the principal extracts as much surplus as possible from the stronger players until they are indifferent between exerting effort and imitating the weakest player to exert no effort. This is optimal because the weakest player is more than three times less productive than the second weakest player. So, it is not worth it to transfer some prize from Player 2 to Player 3 even though it relaxes the incentive compatibility constraints and thus increases the effort of all three players.

The second line shows the revenue that occurs when we transfer half of Player 2's prize to Player 1. This action is optimal because Player 3 is sufficiently productive relative to Player 2 that it is worth it to transfer this prize in order to relax the incentive compatibility constraint of Player 1.

The third line shows the revenue that occurs when we also transfer half of Player 1's prize to Player 3. This gives Player 3 a prize of one. This is not always possible because the individual rationality constraint of Player 2 (and even Player 1) could bind. The condition that guarantees that these constraints do not bind is $\frac{k_3}{k_2} \geq 2$.

The fourth and fifth line are similar to the third but take effect when transferring half of Player 1's prize to Player 3 is not feasible because of Player 2's individual rationality constraint. In the fourth line, some of Player 1's prize is transferred to Player 3 until the individual rationality constraint of Player 2 binds. In the fifth line, more of Player 1's prize is transferred, but this time it must be shared between Player 2 and Player 3 in order to relax the individual rationality constraint of Player 2.

5 Conclusion

In a mechanism design context, anonymity is implicit. The principal must elicit the types of players. In mechanism design, the mechanism must be anonymous because the designer does not know the types of the players. Knowing the types of the players would defeat the purpose of mechanism design.

In contrast, efficiency is a very restrictive condition in a general mechanism design setting. It stipulates that the designer must allocate the entire budget even when he receives the report that both players are low types. In contests, efficiency is almost always implicit. All standard contest forms are efficient and most generalized models of contests involve a contest success function that always allocates the prize.

This paper shows that when the principal knows the interim type distribution, the combination of anonymity and efficiency meaningfully constrains the principal's ability to extract surplus so long as there is one less prize than there are players. In the case where there are fewer than $n-1$ prizes, efficiency and anonymity are not enough to constrain the principal. This is because efficiency is a weaker assumption when there are fewer prizes that must be allocated.

There are likely alternative assumptions that, when combined with anonymity and efficiency, also constrain the principal in a setting with fewer prizes. The critical feature is that it must dampen the designer's ability to collectively punish both a defector and the player who was imitated.

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A Proofs

A.1 Proofs of Proposition 2 and Proposition 1

Proposition 1. *If there are n players, $n-1$ prizes, and $\frac{k_n}{k_{n-1}} \geq n$, then in the optimal anonymous, efficient mechanism, the weakest player exerts no effort and the principal obtains revenue*

$$\sum_{i=1}^{n-1} \frac{1}{2k_i}.$$

Proposition 2. *If there are n players, $n-1$ prizes, and $\frac{k_n}{k_{n-1}} \leq n$, then in the optimal anonymous, efficient mechanism, all players exert effort, the principal obtains revenue less than*

$$\frac{1}{k_n} + \sum_{i=1}^{n-2} \frac{1}{k_i},$$

and at least the two weakest players exert the same effort at the bottom.

Proof. The revelation principle applies here. So, we can restrict attention to mechanisms where the principal asks players for their types and allocated prizes and effort. In addition, by standard arguments, the equilibrium of the optimal mechanism will be monotone in the sense that stronger players provide weakly more effort.

When a player defects and imitates another player’s type, the principal knows that a defection has occurred and can punish the defector by allocating prizes with certainty to all players other than the two who reported the same type. Because of efficiency and anonymity, the principal must award the remaining prize to each of the two remaining players with probability one half. Therefore, the prize of a defector is constant regardless of who is imitating whom. So, imitating player n dominates imitating any other player.

Therefore, the relevant incentive compatibility constraint for players 1 through $n-1$ is:

$$p_i - k_i s_i \geq \frac{1}{2} - k_i s_n.$$

The relevant constraint for Player n is her individual rationality constraint,

$$p_n \geq k_n s_n.$$

There are no other relevant constraints. Clearly, these n constraints are binding.

This immediately pins down the equilibrium when no effort is demanded from Player n. In this case, player n is not allocated a prize and the other players are allocated prizes with probability one. Therefore, $s_i = \frac{1}{2k_i}$ for all $i < n$ and $s_n = 0$. In this case, the revenue is

$$\sum_{i=1}^{n-1} \frac{1}{2k_i},$$

as in Proposition 1.

Increasing the effort demanded from Player n has two effects. On the one hand, it increases revenue directly and by relaxing the incentive compatibility constraints of all other players. On the other hand, it reduces revenue because the principal must allocate a share of the prize to Player n who is less productive than the other players. The overall slope of this effect is $n - \frac{k_n}{k_i}$ where i is the player that Player n's prize comes from. Of course, the cheapest source is to take the prize from Player $n-1$. So, the principal will demand effort from Player n if and only if $n > \frac{k_n}{k_{n-1}}$. This concludes the proof of Proposition 1.

Because the slope does not depend on s_n or p_n, when $n > \frac{k_n}{k_{n-1}}$, the principal will take as much of Player $n-1$'s prize as possible. This is the point where each received the prize with probability one half, and both exert the same effort. In this case, the revenue is

$$\frac{n}{2k_n} + \sum_{i=1}^{n-2} \frac{1}{2k_i}.$$

If the principal wants to obtain even more effort from the weakest player, she can do so by also transferring some of the prize from Player $n-2$ to Player n. This will happen if $n > \frac{k_n}{k_{n-2}}$ and continues either until Player n wins with probability one or the individual rationality constraint for Player $n-1$ is binding. If the individual rationality constrain for Player $n-1$ does not bind, the revenue is given by

$$\frac{n}{k_n} + \sum_{i=1}^{n-3} \frac{1}{2k_i}.$$

This concludes the proof of Proposition 2. □

A.2 Proof of Theorem 3

Theorem 3. *If there are three players and two prizes, the optimal anonymous, efficient mechanism obtains revenue*

$$\begin{cases} \frac{1}{2k_1} + \frac{1}{2k_2} & \text{if } \frac{k_3}{k_2} \geq 3 \\ \frac{1}{2k_1} + \frac{3}{2k_3} & \text{if } \frac{k_3}{k_2} \leq 3 \leq \frac{k_3}{k_1} \\ \frac{3}{k_3} & \text{if } \frac{k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \geq 2 \\ \frac{1}{k_1} + \frac{3-k_3/k_1}{2k_2} & \text{if } \frac{k_3}{k_1} \leq 3 \leq \frac{k_2+k_3}{k_1} \text{ and } \frac{k_3}{k_2} \leq 2 \\ \frac{6-\frac{k_2+k_3}{k_1}}{2k_1} & \text{if } \frac{k_2+k_3}{k_1} \leq 3 \text{ and } \frac{k_3}{k_2} \leq 2. \end{cases}$$

Proof. The first three lines of the conditional statement are given by the proof of Proposition 2 and Proposition 1. The last two lines define what happens when the individual rationality constraint of Player 2 is binding. In this case, $p_2 = k_2 s_2$. The question is whether $p_2 > \frac{1}{2}$. This is optimal if and only if $3 \geq \frac{k_3+k_2}{k_1}$ because the cost of increasing Player 3's effort is that the principal must increase the effort of both Player 2 and Player 3.

If this is not worthwhile, $p_2 = \frac{1}{2}$ which means that $s_3 = s_2 = \frac{1}{2k_2}$. Then, by binding individual rationality of Player 3,

$$p_3 = k_3 s_3 = \frac{k_3}{2k_2}.$$

Then, by efficiency,

$$
\begin{aligned}
p_1 &= 2 - p_2 - p_3 \\
&= 2 - \frac{1}{2} - \frac{k_3}{2k_2} \\
&= \frac{3k_2 - k_3}{2k_2}.
\end{aligned}
$$

We can use the incentive compatibility constraint of Player 1 to solve for s_1:

$$
\begin{aligned}
s_1 &= \frac{p_1}{k_1} - \frac{1}{2k_1} + s_3 \\
&= \frac{3k_2 - k_3}{2k_1k_2} - \frac{1}{2k_1} + \frac{1}{2k_2} \\
&= \frac{1}{k_1} + \frac{1 - {}^{k_3}/_{k_1}}{2k_2}.
\end{aligned}
$$

Then, the sum of the scores is the same as the fourth line of the conditional statement.

If $3 \geq \frac{k_3+k_2}{k_1}$, then $p_2 > \frac{1}{2}$ and this transfer stops until either $p_1 = \frac{1}{2}$ or the incentive compatibility constraint of Player 1 binds.

It turns out that only the latter is possible. To see this, note that if $p_1 = \frac{1}{2}$, then $s_1 = s_2 = s_3 = \frac{3}{2(k_2+k_3)}$. Then, the individual rationality constraint of Player 1 is

$$
\begin{aligned}
p_1 &\geq k_1 s_1 \\
\frac{1}{2} &\geq \frac{3k_1}{2(k_2 + k_3)} \\
3 &\leq \frac{k_2 + k_3}{k_1}
\end{aligned}
$$

which contradicts the condition except in the indifference case (equality).

Therefore, in this case, the incentive compatibility constraints of all players are binding. □

B Algorithm

Not yet written.