Covert Discrimination in All-Pay Contests

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Motivation: Discrimination

Contestants may differ in terms of their inherent ability to win a contest

- This heterogeneity reduces competitiveness and total effort
- Discrimination in favor of the weaker player may reduce or remove this heterogeneity
- Large literature on biasing contests to benefit weaker players (Holzer and Neumark 2000; Roemer 1998) or to maximize aggregate effort (Brown 2011; Franke et al. 2013)

We consider the problem of a revenue maximizing contest designer.

Motivation: Overt vs covert discrimination

Most discrimination in the literature is *overt* in that the designed contest depends on the identities of the participants. This is undesirable if

1. the designer does not know the identities of contestants
2. overt discrimination is not acceptable (eg. offering a larger bonus to the less productive manager)

We instead consider contests with *covert* discrimination where the only asymmetry in the resulting game is the inherent ability difference of the players (i.e. rules are universal).

Model (1): Setup

- A contest designer creates a symmetric prize, v, that **two players**, $1, 2$, compete for in an all-pay contest with complete information
- Each player submits score s_i at cost $c_i(s_i)$
- Player i receives payoff:

$$u_i(s_i; s_{-i}) = p(s_i; s_{-i})v(s_i; s_{-i}) - c_i(s_i)$$

where

$$p(s_i; s_{-i}) = \begin{cases} 1 & \text{if } s_i > s_{-i} \\ 0.5 & \text{if } s_i = s_{-i} \\ 0 & \text{if } s_i < s_{-i} \end{cases}$$

Model (2): Timing

The stages of the previously stated model occur in the following order:

1. Both players made aware of their own cost c_i and the cost of the other c_{-i}. The designer is aware of the cost functions.
2. The contest designer announces a prize, v, to both players.
3. The two players submit their scores (s_1, s_2) simultaneously.
4. The player with the greater score wins and both players become aware of the other's score.
5. Ex-post payoff of player (i) who receives the prize is $v(s_i; s_{-i}) - c_i(s_i)$. The other player (k) gets $-c_k(s_{-i})$.

Model (3): Restrictions

The principal wants to solve

$$\max E[c_1(s_1) + c_2(s_2)]$$

subject to constraint R1 and technical restriction R2.

R1) **Constrained** the expected prize is constrained in equilibrium. That is, $EV := E[p(s_1; s_2)v(s_1; s_2) + p(s_2; s_1)v(s_2; s_1)] \leq \bar{v}$.

R2) **Equilibrium** there exists a threshold, $\bar{s}$, such that the unique equilibrium is in mixed strategies with full support on $[0, \bar{s}]$.[1]

The principal will extract all surplus. So, R2 isn't restrictive in general.

[1] Sufficient conditions are in Betto and Thomas 2021 and repeated on Slide 17.

Model (4): Cost assumptions

WLOG, $c_i(0) = 0$. We make the following assumptions about costs

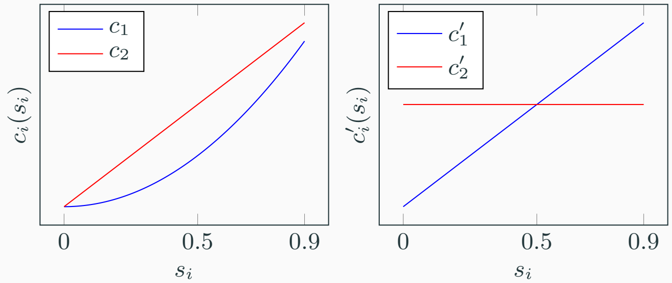
C1) **Monotonicity** $c_i'(s) > 0$, for all $s > 0$ and $i \in \{1, 2\}$.

C2) **Interiority** $\lim_{s\to\infty} c_i(s) = \infty$ for all $i \in \{1, 2\}$.

C3) **Ranked costs** $c_1(s) < c_2(s)$ for all $s > 0$.

C4) **Unranked marginal costs** $\exists t$ s.t. $c_2'(t) < c_1'(t)$ and $\bar{v} \geq c_1(t) + c_2(t)$.

For example, suppose the stronger player has an easier start. However, the weaker player has more ability once she gets past the beginning.



The expected revenue can conveniently be expressed in terms of the expected prize value, $EV \in (0, \bar{v}]$, and the players' payoffs $(\bar{u}_i)$.

$$\begin{aligned} R &= E[c_1(s_1) + c_2(s_2)] = \int_0^{\bar{s}} c_1(y) dG_1(y) + \int_0^{\bar{s}} c_2(y) dG_2(y) \\ &= \int_0^{\bar{s}} \int_0^{y} v(y;x) dG_2(x) dG_1(y) + \int_0^{\bar{s}} \int_0^{y} v(y;x) dG_1(x) dG_2(y) - \bar{u}_1 - \bar{u}_2 \\ &= E[p_i(s_i; s_{-i}) v(s_i; s_{-i}) + p_i(s_{-i}; s_i) v(s_{-i}; s_i)] - \bar{u}_1 - \bar{u}_2 \\ &= EV - \bar{u}_1 - \bar{u}_2 \leq \bar{v}. \end{aligned}$$

where the second line comes from the indifference condition.

Optimal contest without spillovers

Suppose that the principal chooses a prize that depends only on the score of the *winner*. That is, $v(s; y) := \hat{v}(s)$. This case is considered by Jönsson and Schmutzler 2013.

Then, Player 2 always has a payoff of zero. So,

$$\begin{aligned} R &= EV - \bar{u}_1 - \bar{u}_2 \\ &= EV - \bar{u}_1 \\ &= EV - (v(\bar{s}) - c_1(\bar{s})) \\ &= EV - (c_2(\bar{s}) - c_1(\bar{s})) < \bar{v} \end{aligned}$$

Therefore, full surplus extraction is impossible.

Optimal contest with spillovers

Suppose that the principal chooses a prize that depends only on the score of the *loser*. That is, $v(s; y) := \check{v}(y)$. In this setting, **full surplus extraction is possible**.

Theorem 1. There exists a prize $\check{v}^{\star}$ such that $R = \bar{v}$.

- Expected revenue cannot exceed $\bar{v}$. So, this contest is optimal.
- Both players have zero payoffs.
- The expected prize value for player i when she obtains a score of $\bar{s}$ is $c_i(\bar{s})$. Thus, Player 2 obtains a larger expected prize in equilibrium – as in overt discrimination.

Intuition (1)

The continuous portion of equilibrium strategy densities are:

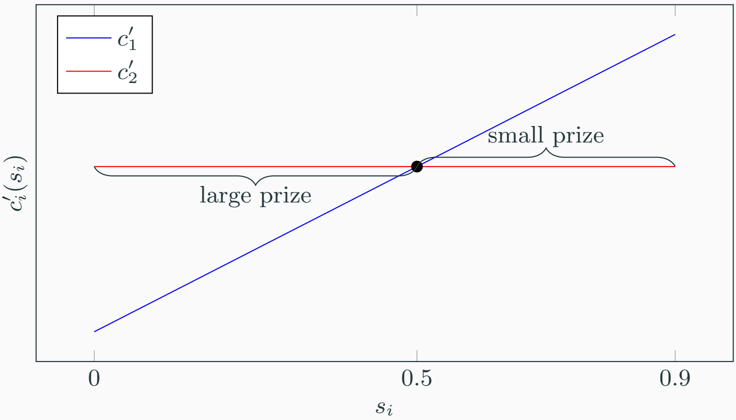
$$g_1(x) = \frac{c_2'(x)}{\check{v}(x)} \qquad g_2(x) = \frac{c_1'(x)}{\check{v}(x)}$$

Thus, $g_i(s) > g_{-i}(s)$ if and only if $c_{-i}'(s) > c_i'(s)$. That is, players put more density on the regions where they have a *marginal advantage*.

Now, suppose the prize value is lower on an interval where Player 2 has a marginal advantage.

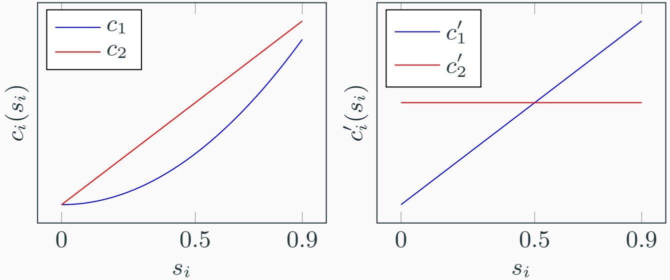
Intuition (2)

Recall that a player's prize depends on her opponent's action and that Player 2 is more likely to place bids that devalue the prize for her opponent.



Example (1)

Suppose $\bar{v} = 0.9$ and that $c_1(x) = x^2$ and $c_2(x) = x$ for all $x \in [0, 0.9]$.[2]

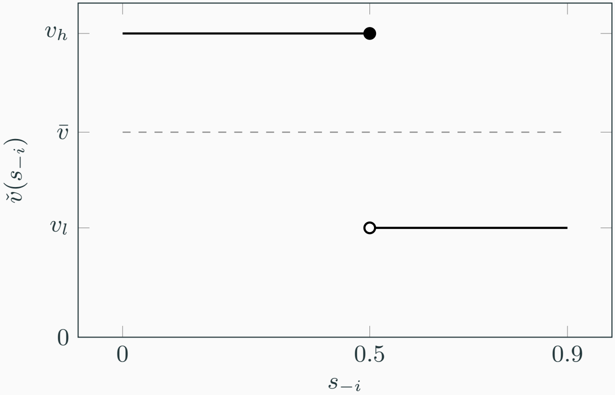


Note that Player 1 has a *marginal advantage* for bids on the interval $[0, 0.5)$ while Player 2 has a marginal advantage for bids on $(0.5, 0.9]$.

[2] We don't use values above 0.9 in this example.

Example: Intuition (2)

As we know, $g_2(s) > g_1(s)$ if and only if $s > 0.5$. Suppose the designer chooses a prize value that is a step function with a lower value on $(0.5, 0.9]$.



Example: Optimal simple contest (3)

Finding an optimal contest involves satisfying these three equations

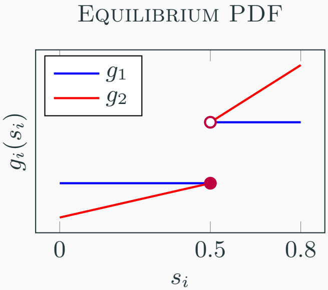
$$E[c_1(s_1) + c_2(s_2)] = \bar{v} \qquad \int_0^{\bar{s}} g_1(x)\,dx = 1 \qquad \int_0^{\bar{s}} g_2(x)\,dx = 1$$

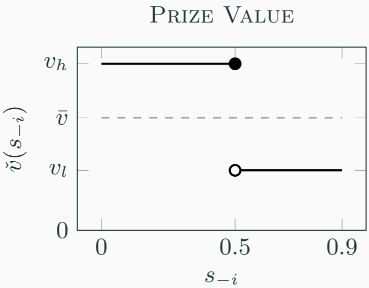
which, with the aforementioned step function are equivalent to

$$\frac{1}{v_l} c_1(\bar{s}) c_2(\bar{s}) - \left(\frac{1}{v_l} - \frac{1}{v_h}\right) c_1(0.5) c_2(0.5) = 0.9$$
$$\frac{1}{v_l} c_2(\bar{s}) - \left(\frac{1}{v_l} - \frac{1}{v_h}\right) c_2(0.5) = 1$$
$$\frac{1}{v_l} c_1(\bar{s}) - \left(\frac{1}{v_l} - \frac{1}{v_h}\right) c_1(0.5) = 1.$$

We solve these three equations for $v_l, v_h, \bar{s}$ to get $v_l = \frac{12}{25}$, $v_h = \frac{4}{3}$, and $\bar{s} = \frac{4}{5}$.

Example: Optimal simple contest (4)





Note that Player 2 is more likely to place bids that make her opponent's prize worth v_l and Player 1 is more likely to place bids that make her opponent's prize worth v_h.

Maximal discrimination (preliminary)

The principal could discriminate more by reducing v_l and increasing v_h.

- This would not be revenue maximizing.
- Player 2 can receive a positive payoff.

We can show that any

$$\bar{u}_2 < \bar{v} - \lim_{\bar{s} \to 0.5} \frac{c_1(\bar{s})c_2(\bar{s}) - c_1(0.5)c_2(0.5)}{c_1(\bar{s}) - c_1(0.5)} = 0.15$$

is achievable with $v_h \to \infty$, $v_l \to 0$, and $\bar{s} \to 0.5$.

Appendix

Sufficient Conditions

By Betto and Thomas 2021, the following are sufficient for equilibrium uniqueness and for the strategies to have support on $[0, \bar{s}]$ for some $\bar{s} > 0$.

A1 **Smoothness** $v(s; y)$ and $c_i(s)$ are continuously differentiable in s (own score) for all $s \geq 0$ and $v(s; y)$ is continuous in y (other score).

A2 **Monotonicity** for every $s \geq 0$, $v'(s; y) < c_i'(s)$ holds a.e. where $v'(s; y) \equiv \frac{\partial v(s;y)}{\partial s}$.

A3 **Interiority** $v(0, 0) > c_i(0) = 0$ and $\lim_{s_i \to \infty} \max_y v(s_i; y) < c(s_i)$.

A4 **Discontinuity at ties** just winning with score s is better than losing with score s_i. That is, $v(s; s) > 0$.

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