Covert Discrimination in All-pay Contests

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Abstract

A contest designer may wish to disadvantage a stronger player to improve competitiveness. We show this can be done in all-pay auctions such that the game is fair (i.e. symmetric) ex-ante. Yet, the stronger player is endogenously offered a lower prize in expectation. Thus, discrimination is *covert*.

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1 Introduction

Many economic settings are described by competition between individuals for a prize. However, these competitions can be asymmetric in the sense that contestants may differ in their inherent ability to win the contest. This heterogeneity can reduce competitiveness and decrease effort. When this happens, a contest designer may want to level the playing field by discriminating in favor of the weaker player. This approach has been explored by the large empirical and theoretical literature on discrimination as a way of benefiting weaker players (Holzer and Neumark 2000; Roemer 1998) and maximizing aggregate effort (Brown 2011; Franke et al. 2013).

Most of the discrimination considered in the contest design literature is *overt* – that is, the designer makes the contest depend on the identities of the participants. For example, Mealem and Nitzan 2014 studies optimal reward schemes in a complete information all-pay auction (APA) where the stronger player is offered a smaller reward for winning than her opponent. These methods require both that the designer knows the identities of the contestants and is allowed to design a contest that treats players asymmetrically. In practice, it may not be viable – or even desirable – to construct a contest that overtly favors one player. This could be due to concerns about fairness. For example, an employer may be unable to offer larger bonuses to less productive employees. It could also be due to information. For example, the designer may not be able to identify the weaker player.

We instead consider discrimination that is *covert* in the sense that neither the value nor the allocation of the prize depends on the identities of the players.[1] Indeed, the only asymmetries are the players' inherent differences in ability, i.e. their cost functions. Such schemes have the advantage of apparent fairness, since players are treated symmetrically. In order to achieve discrimination in this setting, the key is to make sure stronger players are still disadvantaged *in equilibrium*.

We achieve this in an APA with a symmetric endogenous reward scheme that depends on the players' opponents' effort. We show that covert discrimination is possible if marginal costs are not ranked. If this condition is met, full surplus extraction is possible. This implies that a designer can always completely level the playing field. We also show that these endogenous reward schemes can generate more total revenue than any exogenous one.

This paper contributes to the literature on optimal design of all-pay contests (eg. Che and Gale 2003; Mealem and Nitzan 2014) by allowing the designer to introduce spillovers to contests. We especially relate to Jönsson and Schmutzler 2013, which studies the design of

[1] Nti 2004 study covert discrimination in Tullock contests where a contest designer can adjust the marginal return to effort in the contest success function to maximize competitiveness. This is impossible in an all-pay contest where the relationship between effort and prize allocation is deterministic.

all-pay contests with endogenous prizes. Unlike their paper, we study contests with spillovers where the prize depends on the opponents' effort rather than the players' own. This difference is crucial and drives our results.

We also contribute to the literature on optimal covert discrimination in Tullock contests (eg. Epstein et al. 2013; Nti 1999), which studies how the contest success function (CSF) can be adjusted (symmetrically) to maximize revenue. In contrast, we hold the CSF constant (the all-pay CSF, in particular), but adjust the prize (symmetrically) to maximize revenue.

2 Model

We focus on two player asymmetric all-pay contests with spillovers as analyzed in Betto and Thomas 2021. Each agent selects actions $s_i \in [0, \infty)$ and receives the following payoff

$$u_i(s_i; s_{-i}) \equiv p(s_i; s_{-i})v(s_i; s_{-i}) - c_i(s_i)$$

where v is a symmetric prize designed by the principal and c_i is the player's inherent cost. The prize goes to the player with the larger score. That is, allocation of the prize is defined by $p(s_i; s_{-i})$ where

$$p(s_i; s_{-i}) = \begin{cases} 1 & \text{if } s_i > s_{-i} \\ 0.5 & \text{if } s_i = s_{-i} \\ 0 & \text{if } s_i < s_{-i} \end{cases}$$

Note that while the players differ in their ability produce scores that enable them to win the contest, the two players are equally adept at producing effort that the contest designer values. That is, the return to the designer from each Player i is $E[c_i(s_i)]$.

The principal's objective is to maximize the expected revenue of the auction subject to a constraint on the expected value of the prize ($\bar{v}$). That is, maximize

$$\max E[c_1(s_1) + c_2(s_2)] \text{ s.t. } EV \leq \bar{v}$$

where $EV \equiv E[p(s_1; s_2)v(s_1; s_2) + p(s_2; s_1)v(s_2; s_1)]$.

Note that the value of the prize can depend on the actions of both players. This possible dependence on the score of the losing player (s_{-i}) is known as a spillover. Allowing the principal to construct prizes with spillovers will allow for full surplus extraction.

2.1 Assumptions

The allowable c_i are restricted by the following technical assumptions.

Assumption 1 (A1, Monotonicity). *$c_i'(s) > 0$, for all $s > 0$ and $i \in \{1, 2\}$.*

Assumption 2 (A2, Interiority). *For all $i \in I$,*

$$\lim_{s \to \infty} c_i(s) = \infty$$

Assumption 3 (A3, Ranked costs). *For all $s > 0$, $c_1(s) < c_2(s)$.*

Assumption 4 (A4, Unranked marginal costs). *There exists a t such that $c_2'(t) < c_1'(t)$ and $\bar{v} \geq c_1(t) + c_2(t)$.*

Assumptions A3 and A4 motivate the setting of discrimination. They say that costs are ranked, such that Player 1 has an absolute advantage over Player 2. It also ensures that Player 2 has a comparative advantage on some interval. That is, there are some bids for which Player 2 has a lower marginal cost. This assumption drives our results.

3 Results

Whatever the equilibrium of the contest, the following condition must hold for all x in the support of Player i's strategy.

$$\bar{u}_i \equiv E[p(x; s_{-i})v(x; s_{-i})] - c_i(x) \tag{1}$$

where $\bar{u}_i$ is the payoff of Player i and s_{-i} is a (possibly deterministic) random variable representing the equilibrium strategy of Player i's opponent. This implies that the principal's expected revenue can be rewritten as follows:

$$\begin{aligned} E[R] &= E[c_1(s_1) + c_2(s_2)] = E[p(s_1; s_2)v(s_1; s_2) + p(s_2; s_1)v(s_2; s_1)] - \bar{u}_1 - \bar{u}_2 \\ &= EV - \bar{u}_1 - \bar{u}_2. \end{aligned}$$

When the prize limit is binding, $EV = \bar{v}$. So, subject to this constraint, the game is constant sum and the principal's problem is equivalent to minimizing the payoffs of the agents.

3.1 All-pay contests without spillovers

Suppose that the principal chooses a prize that depends only on the score of the winner. That is, $v(s;y) := \hat{v}(s)$. This is the design case considered by (Jönsson and Schmutzler 2013).

Under the assumptions of Siegel 2009, full surplus extraction is not possible in an APA without spillovers. In fact, in a two player all-pay contest without spillovers, Player 2 has a payoff of zero and Player 1 has a payoff of

$$\bar{u}_1 = c_2(\bar{s}) - c_1(\bar{s}) > 0$$

where $\bar{s}$ solves $c_2(\bar{s}) = v(\bar{s})$.

3.2 All-pay auctions with spillovers

When the designer can construct a prize with spillovers, full rent extraction is possible.

Theorem 1 (Full surplus extraction)**.** *There exists an optimal contest of the form* $v(s;y) = \check{v}^{\star}(y)$ *such that the expected revenue from the contest is* $\bar{v}$*.*

Theorem 1 shows that it is possible for the principal to achieve the first-best outcome with spillovers. The prize does not even have to depend on the player's own score. The optimal prize is easily constructed. The function is a continuous approximation of a step function that is larger than $\bar{v}$ on the region where $c_2'(s) > c_1'(s)$, but is smaller in the region where $c_2'(s) < c_1'(s)$.

Example 1. To exemplify how covert discrimination works in an all-pay framework, consider the following example where two players compete for a prize. The player who produces the largest value of an observable output is declared the winner. The players however differ in their ability to produce such output: Player 1 produces output s at cost $c_1(s) = s^2$, whereas Player 2 can produce the same amount at cost $c_2(s) = s$.

Suppose that the contest designer is able to promise a prize of v to the winner as long as $EV \leq 0.9$ – i.e. the contest designer has a budget she must not exceed in expectation. Note that on the interval $(0, 0.9]$, $c_2(s) > c_1(s)$ but marginal costs are not similarly ranked. Because of this, there exists a prize such that neither player has a mass point at zero.

The fixed prize that maximizes effort, given the designer's constraint, is $v = 0.9$. In this case, the players' equilibrium strategies' cumulative distribution functions are

$$G_1(s) = \frac{10s}{9} \qquad G_2(s) = \frac{1}{10} + \frac{10s^2}{9}.$$

The expected output produced by both competitors is $0.9 - (0.9 - (0.9)^2) = 0.81$. Now, consider the following prize:

$$\check{v}(x) = \begin{cases} \frac{4}{3} & \text{if } x \leq \frac{1}{2} \\ \frac{11}{25} & \text{if } x > \frac{1}{2} \end{cases}$$

where x is the opponent's output. That is, the value of the prize to one player depends explicitly on the output produced by their opponent. With this particular reward choice, $EV = 0.9$ and total output is 0.9.

This example is visualized in Figure 1, and demonstrates that covert discrimination can be revenue improving. The key to this example lies in how the stronger player 1's marginal costs are above player 2's for $s > 1/2$, even if the costs themselves aren't. By carefully crafting the prize values, the designer is able to exploit the marginal disadvantage of the stronger player to improve the contests' competitiveness without having to rely on differential treatment of the participants. △

4 Appendix

4.1 Proof of Theorem 1

Define the sets L, H as $L \equiv \{x \in [0, \bar{v}] | c_1'(x) > c_2'(x)\}$ and $H \equiv [0, \infty) - L = \{x \in [0, \infty) | c_1'(x) \leq c_2'(x)\}$. We propose the following step contest and show that $R = \bar{v}$

$$\check{v}^{\star}(x) = \begin{cases} v_l & \text{if } x \in L \\ v_h & \text{if } x \in H \end{cases}$$

for some $v_h \geq v_l > 0$. That is, we need to show that there exist $v_h \geq v_l > 0$ such that

$$R \equiv \int_0^{\bar{s}} c_1(x)\, dG_1(x) + \int_0^{\bar{s}} c_2(x)\, dG_2(x) = \bar{v} \tag{2}$$

where $\bar{s}, G_1, G_2$ are equilibrium objects. When there is no mass point in either player's strategy distribution,

$$g_i(x) = \frac{c_{-i}'(x)}{\check{v}^{\star}(x)} = \begin{cases} \frac{c_{-i}'(x)}{v_l} & \text{if } x \in L \\ \frac{c_{-i}'(x)}{v_h} & \text{if } x \in H. \end{cases}$$

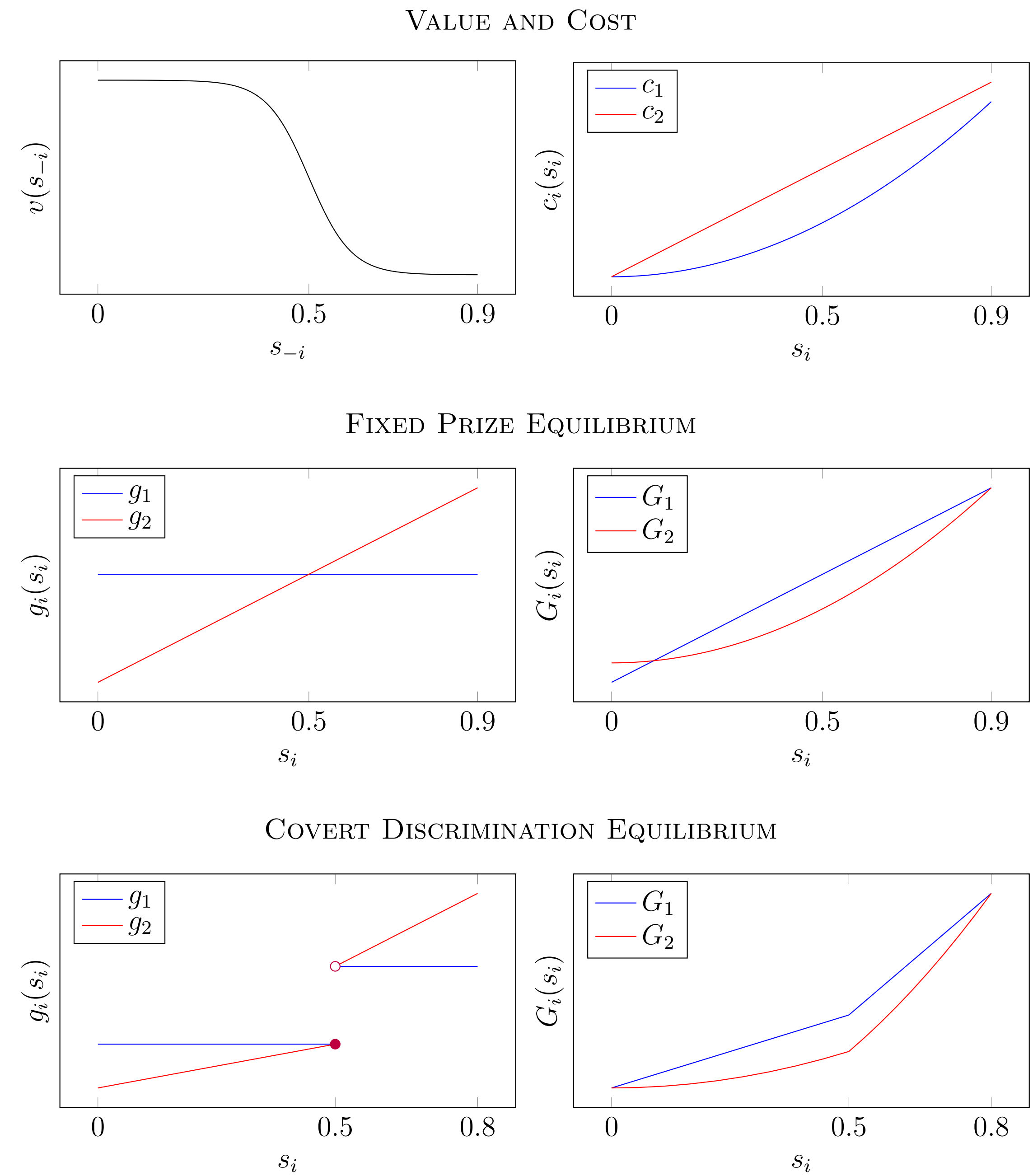

Figure 1: Plots of equilibrium densities (left), distributions (right), and primitives (top). In both cases, Player 2 places more density on scores in $(0.5, 0.9]$ than Player 1. Because the prize is sharply less valuable when your opponent plays in this interval, in equilibrium, the prize is not as valuable for Player 1. This drives the increase in the contests' competitiveness.

There are no mass points if and only if $\int_0^{\bar{s}} g_1(x)\,dx = \int_0^{\bar{s}} g_2(x)\,dx = 1$. In this case, this is equivalent to

$$
\begin{aligned}
G_{-i}(\bar{s}) &= \int_0^{\bar{s}} g_{-i}(x)\,dx \\
&= \int_0^{\bar{s}} \frac{c_i'(x)}{\check{v}^\star(x)}\,dx \\
&= \frac{1}{v_l}\int_{L\cap S} c_i'(x)\,dx + \frac{1}{v_h}\int_{H\cap S} c_i'(x)\,dx \\
&= \frac{1}{v_l}c_i(\bar{s}) - \left(\frac{1}{v_l} - \frac{1}{v_h}\right)\int_{H\cap S} c_i'(x)\,dx = 1
\end{aligned} \tag{3}
$$

for both i.

In this case, equation (2) can be rewritten as

$$
\begin{aligned}
R &\equiv \int_0^{\bar{s}} c_1(x)\,dG_1(x) + \int_0^{\bar{s}} c_2(x)\,dG_2(x) \\
&= \int_0^{\bar{s}} \frac{1}{\check{v}^\star(x)}\left(c_1(x)c_2'(x) + c_1'(x)c_2(x)\right)\,dx \\
&= \frac{1}{v_l}\int_{L\cap S} \frac{dc_1(x)c_2(x)}{dx}\,dx + \frac{1}{v_h}\int_{H\cap S} \frac{dc_1(x)c_2(x)}{dx}\,dx \\
&= \frac{1}{v_l}c_1(\bar{s})c_2(\bar{s}) - \left(\frac{1}{v_l} - \frac{1}{v_h}\right)\int_{H\cap S} \frac{dc_1(x)c_2(x)}{dx}\,dx = \bar{v}.
\end{aligned} \tag{4}
$$

If we substitute $l \equiv \frac{1}{v_l}$ and $h \equiv \frac{1}{v_l} - \frac{1}{v_h}$ into (3) and (2), we get the following system of equations

$$
\begin{aligned}
\bar{v} &= lc_1(\bar{s})c_2(\bar{s}) - h\int_{H\cap S} \frac{dc_1(x)c_2(x)}{dx}\,dx \\
1 &= lc_1(\bar{s}) - h\int_{H\cap S} c_1'(x)\,dx \\
1 &= lc_2(\bar{s}) - h\int_{H\cap S} c_2'(x)\,dx
\end{aligned}
$$

We can isolate l, h from the second and third equation

$$
l = \frac{\int_{H\cap S} c_2'(x) - c_1'(x)\,dx}{D(\bar{s})} > \frac{c_2(\bar{s}) - c_1(\bar{s})}{D(\bar{s})} = h
$$

where

$$
D(\bar{s}) \equiv c_2(\bar{s})\int_{L\cap S} c_1'(x)\,dx - c_1(\bar{s})\int_{L\cap S} c_2'(x)\,dx > 0.
$$

If an $\bar{s}$ exists, it is obvious that $\bar{s} > k \equiv \inf L$. Otherwise, none of these equations are defined. So, the only thing to confirm is that there exists a $\bar{s} > k$ such that

$$Q(\bar{s}) \equiv \frac{c_1(\bar{s})c_2(\bar{s}) \int_{H\cap S} c_2'(x) - c_1'(x)\,dx - (c_2(\bar{s}) - c_1(\bar{s})) \int_{H\cap S} \frac{dc_1(x)c_2(x)}{dx}\,dx}{c_1(\bar{s}) \int_{H\cap S} c_2'(x)\,dx - c_2(\bar{s}) \int_{H\cap S} c_1'(x)\,dx} = \bar{v}$$

We know that $c_1'(k) = c_2'(k)$.

$$\lim_{x\to k} Q(x) = \lim_{x\to k} \frac{c_1(x)c_2(x) \int_{H\cap S(x)} c_2'(y) - c_1'(y)\,dy - (c_2(x) - c_1(x)) \int_{H\cap S(x)} \frac{dc_1(y)c_2(y)}{dy}\,dy}{c_1(x) \int_{H\cap S(x)} c_2'(y)\,dy - c_2(x) \int_{H\cap S(x)} c_1'(y)\,dy}$$

By L'Hopital's rule

$$\begin{aligned}
&= \frac{\frac{dc_1(k)c_2(k)}{dk}(c_2(k) - c_1(k)) - (c_2'(k) - c_1'(k))\, c_1(k)c_2(k)}{c_1'(k)c_2(k) - c_1(k)c'2(k)} \\
&= \left(\frac{c_1'(k)c_2(k) + c_1(k)c_2'(k)}{c_1'(k)c_2(k) - c_1(k)c_2'(k)}\right)(c_2(k) - c_1(k)) \\
&= c_1(k) + c_2(k)
\end{aligned}$$

Additionally,

$$\begin{aligned}
Q(x) &= \frac{c_1(x)c_2(x) \int_{H\cap S(x)} c_2'(y) - c_1'(y)\,dy - (c_2(x) - c_1(x)) \int_{H\cap S(x)} \frac{dc_1(y)c_2(y)}{dy}\,dy}{c_1(x) \int_{H\cap S(x)} c_2'(y)\,dy - c_2(x) \int_{H\cap S(x)} c_1'(y)\,dy} \\
&> \frac{c_1(x)c_2(x) \int_{H\cap S(x)} c_2'(y) - c_1'(y)\,dy - (c_2(x) - c_1(x))\, c_1(x)c_2(x)}{c_1(x) \int_{H\cap S(x)} c_2'(y)\,dy - c_2(x) \int_{H\cap S(x)} c_1'(y)\,dy} \\
&= \frac{c_1(x)c_2(x) \int_{L\cap S(x)} c_1'(y) - c_2'(y)\,dy}{c_1(x) \int_{H\cap S(x)} c_2'(y)\,dy - c_2(x) \int_{H\cap S(x)} c_1'(y)\,dy} \\
&> \frac{c_1(x)c_2(x) \int_{L\cap S(x)} c_1'(y) - c_2'(y)\,dy}{c_1(x)c_2(x) - c_2(x) \int_{H\cap S(x)} c_1'(y)\,dy} \\
&= \frac{c_1(x) \int_{L\cap S(x)} c_1'(y) - c_2'(y)\,dy}{\int_{L\cap S(x)} c_1'(y)\,dy} \\
&= c_1(x)\left(1 - \frac{\int_{L\cap S(x)} c_2'(y)\,dy}{\int_{L\cap S(x)} c_1'(y)\,dy}\right)
\end{aligned}$$

So, $\lim x \to \infty Q(x) = \infty$. By continuity, there exists a $\bar{s}$ such that $Q(\bar{s}) = \bar{v}$.